Sections 5.4 & 5.5 – I.C.E – Solving Trig Equations

Using any of the above formulas, Find the exact value of the following (show your work):

1. $\cos 15°$ $= \cos(45-30) = \cos 45\cos 30 + \sin 45 \sin 30$

$= \frac{\sqrt{2}}{2}\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\frac{1}{2}$

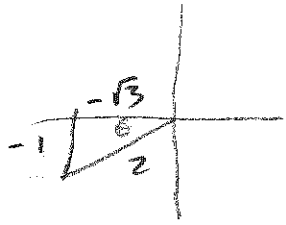
$= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \boxed{\frac{\sqrt{6}+\sqrt{2}}{4}}$

2. $\sin\frac{13\pi}{12}$ $= \sin\frac{\pi}{12} = \sin\frac{\frac{\pi}{6}}{2} = \sin\frac{30}{2}$

Q3 $= \ominus\sqrt{\frac{1-\cos 30}{2}} = -\sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}} = -\sqrt{\frac{2-\sqrt{3}}{4}} = \boxed{-\frac{\sqrt{2-\sqrt{3}}}{2}}$

3. $\tan\frac{17\pi}{12}$ $= \tan 255 = \tan(210+45)$

Q3 $= \frac{\tan 210 + \tan 45}{1-\tan 210\tan 45} =$



$= \frac{\frac{\sqrt{3}}{3}+1}{1-\frac{\sqrt{3}}{3}\cdot 1} = \frac{\frac{\sqrt{3}+3}{3}}{\frac{3-\sqrt{3}}{3}} = \frac{3+\sqrt{3}}{3-\sqrt{3}}\left(\frac{3+\sqrt{3}}{3+\sqrt{3}}\right)$

$= \frac{9+6\sqrt{3}+3}{9-3} = \frac{12+6\sqrt{3}}{6} = \boxed{2+\sqrt{3}}$

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4. Write the following as a **single trigonometric** function:

$2\sin A\cos A =$ $\sin 2A$

$\dfrac{2\tan B}{1-\tan^2 B} =$ $\tan 2B$

$1-2\sin^2 C =$ $\cos 2C$

$2\sin 105^\circ \cos 105^\circ =$ $\sin 210^\circ$

$\cos^2 15^\circ - \sin^2 15^\circ =$ $\cos 30^\circ$

$\dfrac{2\tan 67.5^\circ}{1-\tan^2 67.5^\circ} =$ $\tan 135^\circ$

$2\cos^2 112.5^\circ - 1 =$ $\cos 225^\circ$

$\sin 75^\circ \cos 15^\circ + \sin 15^\circ \cos 75^\circ =$ $\sin(75+15) = \sin 90^\circ$

$\dfrac{\tan 52.5^\circ + \tan 7.5^\circ}{1-\tan 52.5^\circ \tan 7.5^\circ} =$ $\tan(52.5+7.5) = \tan 60^\circ$

$\cos 127.5^\circ \cos 7.5 + \sin 127.5^\circ \sin 7.5^\circ =$ $\cos(127.5-7.5) = \cos 120^\circ$

$\sin\left(\dfrac{\pi}{12}\right)\cos\left(\dfrac{\pi}{12}\right) =$ $\dfrac{\sin\frac{\pi}{6}}{2}$

$\sin 15\cos 15 = \dfrac{\sin 30}{2}$

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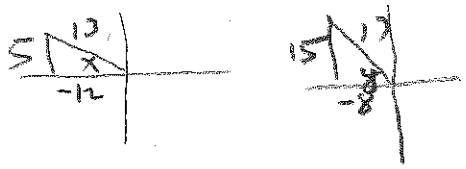
5. Find the exact values of the following. It would be helpful to draw a picture!

$\sin x = \frac{5}{13}$, $\cos y = \frac{-8}{17}$; x & y in QII



$\cos 2x =$ $\cos^2 x - \sin^2 x$

$= \left(\frac{-12}{13}\right)^2 - \left(\frac{5}{13}\right)^2$

$= \frac{144-25}{169} = \frac{119}{169}$

$\tan 2x =$

$\frac{2\tan x}{1-\tan^2 x} = \frac{2\left(\frac{-5}{12}\right)}{1-\left(\frac{-5}{12}\right)^2}$

$= \frac{\frac{-5}{6}}{\frac{144-25}{144}} = \frac{-5}{6} \cdot \frac{144}{119} = \frac{-120}{119}$

$\sin(x+y) =$

$\sin x \cos y + \cos x \sin y$

$= \frac{5}{13} \cdot \frac{-8}{17} + \frac{-12}{13} \cdot \frac{15}{17}$

$= \frac{-40-180}{221} = \frac{-220}{221}$

$\tan(y-x) = \frac{\tan y - \tan x}{1 + \tan y \tan x}$

$= \frac{-\frac{15}{8} - -\frac{5}{12}}{1 + \left(\frac{-15}{8}\right)\left(\frac{-5}{12}\right)} = \frac{\frac{-45+10}{24}}{\frac{96+75}{96}}$

$= \frac{-35}{24} \cdot \frac{96}{171} = \frac{-140}{171}$

6. Matching (Some of the expressions on the right may be used more than once or not at all).

a. $\sin(\alpha-\beta)$ 5	1. $\sin\beta$
b. $\cos(\alpha+\beta)$ 6	2. $\sin\alpha\cos\beta+\cos\alpha\sin\beta$
c. $\sin(180°+\beta)$ 7	3. $-\cos\beta$
d. $\sin(180°-\beta)$ 1	4. $\cos\alpha\cos\beta+\sin\alpha\sin\beta$
e. $\cos(180°+\beta)$ 3	5. $\sin\alpha\cos\beta-\cos\alpha\sin\beta$
f. $\sin(\alpha+\beta)$ 2	6. $\cos\alpha\cos\beta-\sin\alpha\sin\beta$
g. $\cos(90°-\beta)$ 1	7. $-\sin\beta$
h. $\cos(\alpha-\beta)$ 4	8. $\cos\beta$

Sections 5.4 & 5.5 – I.C.E – Solving Trig Equations

Find the **solutions** in the interval $[0, 2\pi)$ (think back to section 5.3)

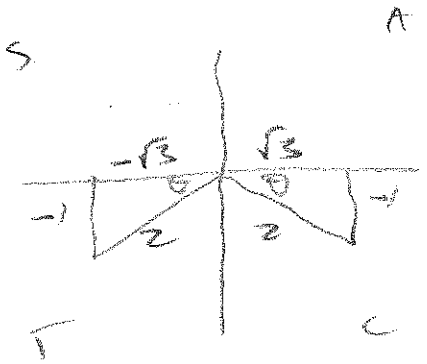
7. $\sin 2x\cos x + \cos 2x\sin x = -\frac{1}{2}$

$= \sin(2x + x) = -\frac{1}{2}$

$\sin 3x = -\frac{1}{2}$

$3x = \frac{7\pi}{6}, \frac{11\pi}{6}$

$\Rightarrow x = \frac{7\pi}{18}, \frac{11\pi}{18}$



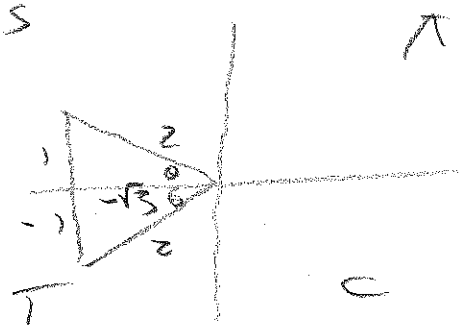
8. $\cos 2x\cos x - \sin 2x\sin x = -\frac{\sqrt{3}}{2}$

$\Rightarrow \cos(2x + x) = -\frac{\sqrt{3}}{2}$

$\cos 3x = -\frac{\sqrt{3}}{2}$

$3x = \frac{5\pi}{6}, \frac{7\pi}{6}$

$\Rightarrow x = \frac{5\pi}{18}, \frac{7\pi}{18}$



9. $\sin 2x + 2\cos x = 0$

$2\sin x\cos x + 2\cos x = 0$

$2\cos x(\sin x + 1) = 0$

$2\cos x = 0$

$\cos x = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

$\sin x + 1 = 0$

$\sin x = -1$

$x = \frac{3\pi}{2}$